Capacitors can radiate: Further results for the two-capacitor problem

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Using a point dipole radiator approximation, we extend the study of Boykin, Hite, and Singh [Am. J. Phys. **70** (4), 415–420 (2002)] by considering electromagnetic radiation from both the capacitors and the wire loop to account for the missing energy in the popular two-capacitor Kirchoff circuit. Through a series of gedanken experiments in which successively more realistic circuit elements are added, we assess the significance of electromagnetic radiation in accounting for all the missing energy. Extensive use is made of an energy partition function to obtain many results without an explicit solution of nonlinear differential equations. However some difficulties remain, which are posed by the required boundary conditions when an inductance L is included. Implications for radio frequency interference, as well as novel antenna designs are also discussed.

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I. INTRODUCTION

In a recent paper, Boykin, Hite, and Singh[1] considered the two-capacitor problem with radiation using an electric current loop model from which they recovered the missing energy in the traditional Kirchoff circuit.[2] The circuit apparently violates energy conservation when one capacitor is suddenly switched to transfer charge to another. This paradox had been discussed previously[3–5] without a quantitative treatment of radiation. Reference 1 brought to light the significance of radiation, which can in certain circumstances account for all the missing energy (with apologies for the pun). In this paper we examine the significance of the radiation further and show explicitly that there is radiation from the capacitors that was not considered in Ref. 1. We show that the neglect of capacitor radiation requires some special arrangements, which could be realized by enclosing each capacitor in a Faraday cage for example, and cannot be automatically obtained by going to a small (as compared to the radius of the wire loop) point dipole limit.

The capacitor dipole radiator, in which the displacement current replaces the electron current of a wire dipole radiation source, is not usually discussed in many standard texts on electromagnetism[6–8] and in particular antenna theory.[9–11] The exception is Schelkunoff,[10,12,13] who highlighted the duality of magnetic dipole radiation due to a physical electric current loop and the electric dipole radiation due to a fictitious magnetic ''charge'' current loop in his treatment.[14,15] Schelkunoff's seminal biconical antenna[12,15] is perhaps one of the few exactly solvable antenna models that also demonstrates capacitor radiation.

The importance of direct radiation from capacitors has important implications for radio frequency interference.[16] It also is an important issue for alternative nonconventional antenna designs, currently hotly debated in both the engineering and amateur radio literature.[17,18] It is further hoped that this paper will help to clarify the basic physics involved in these alternative antennas and to provide a more complete picture of the two-capacitor problem with radiation.[1,3,4]

II. PREVIOUS STUDIES

We first summarize the main results of Boykin, Hite, and Singh and introduce our notation, which conforms as closely as possible to theirs.[1] These authors considered the connecting wires between the two capacitors C_1 and C_2 (see Fig. 1) as an electric current loop from which they derived the radiation power [via the Poynting vector $\mathbf{S}=(\mathbf{E}\times\mathbf{B})/\mu_0$] in the following formulas:

$$P_{I,\mathrm{rad}}=K_I[\ddot{I}(t-r/c)]^2, \tag{1}$$

$$K_I=\frac{\pi b^4}{6\epsilon_0 c^5}, \tag{2}$$

where b is the radius of the current loop and $I(t-r/c)$ is the retarded current of the loop (not necessarily assumed to be sinusoidal). Here ϵ_0 and μ_0 are the permittivity and the permeability of free space, respectively, and we have included a subscript I to denote quantities for the electric current loop. We use the following parameters $C_1=C_2=200\ \mu\mathrm{F}$, $b=5$ cm, $a=0.5$ mm, $\ell=1$ mm, $R_w=1.4\ \mathrm{m}\Omega$ for the usual wire loop to conform with earlier work unless otherwise stated.[1,3] Here a is the radius of the copper wire with wire resistance R_w, and ℓ is the spacing in the capacitor plates.

If we use a lumped-parameter circuit model (see Fig. 1) in which we have explicitly enclosed the capacitors in Faraday cages to exclude radiation, then the nonlinear circuit element X can be included to represent the impedance properties due to radiation loss.[19,20] The voltage drop V_X across X is then given by

$$V_X=\frac{P_{I,\mathrm{rad}}}{I}=K_I\frac{\ddot{I}^2}{I}, \tag{3}$$

and Kirchoff's voltage law for this circuit implies that

$$V_X+V_2-V_1=0, \tag{4}$$

where V_1 and V_2 are the voltages across the capacitors C_1 and C_2, respectively.

If we introduce a parallel capacitance,

$$\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}, \tag{5}$$

and express the voltage across this capacitor C_s as $V_c=V_2-V_1=-V_X$, we obtain the following nonlinear differential

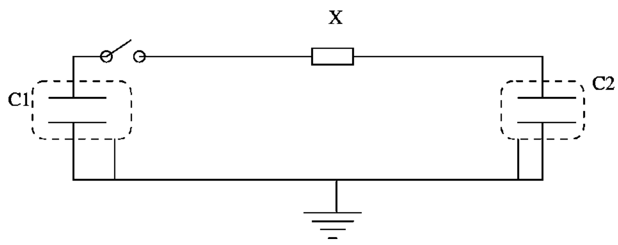

Fig. 1. Lumped-parameter circuit in which X includes the nonlinear elements that account for the radiation properties of the wire loop and/or the capacitors. For the analysis in Ref. 1, we have to enclose the capacitors in Faraday cages (shown dotted) to prevent radiation (see the text).

equation [correcting a misprint in Eq. (12) in Ref. 1] for the circuit:

$$\dddot{V}_c^2 + \frac{1}{K_I C_s}\dot{V}_c V_c = 0. \tag{6}$$

Of particular significance is the result for the equivalent radiation resistance of the loop $V_X = R_{I,\mathrm{rad}} I$ given by[1]

$$R_{I,\mathrm{rad}} = K_I s_1^4 = \frac{K_I^{1/5}}{C_s^{4/5}}, \tag{7}$$

where

$$s_1 = -\left(\frac{1}{K_I C_s}\right)^{1/5}, \tag{8}$$

which we will revisit many times.

There is no doubt that the derivation in Ref. 1 demonstrates that the missing energy can be accounted for entirely by radiation from the wire loop, as they have shown using a magnetic dipole, that is, electric current loop model.[8–11] We now address the question as to what would happen if the current loop is shrunk to an infinitely small radius, or alternatively, the wires are replaced by suitably shielded coaxial transmission lines to prevent radiation. In the model of Ref. 1 (with Faraday shields over the capacitors included), we would then return to the over-idealized circuit, and as has been pointed out, the missing energy paradox remains.[3,4] In this case, we would need to consider wire resistance[2] or self-inductance[3] as the only other sinks for the missing energy. This gedanken experiment shows that the analysis of capacitor radiation is necessary in the two-capacitor problem, although the radiation can be neglected if there are some special arrangements in the experimental configuration. We do this analysis in Sec. III using a point dipole radiator approximation with zero length wire loops and then return to the case for which both capacitors and wire loops are allowed to radiate.

III. OSCILLATING ELECTRIC DIPOLE MODEL

In this gedanken experiment, we remove the Faraday shields on the capacitors and reduce the wire loop to zero length. In the long wavelength ($\lambda \to \infty$) limit, the two capacitors connected by zero length wires can be viewed as an oscillating electric dipole arising from time-varying charges on the parallel plates of C_s. The power intensity of radiation from such a dipole of moment $\mathbf{p}$ is given by[21]

$$P_{C,\mathrm{rad}} = \frac{1}{6\pi\epsilon_0 c^3}|\ddot{\mathbf{p}}|^2. \tag{9}$$

If we consider our ideal point dipole capacitor model as two parallel plates with separation ℓ, we have

$$\ddot{p} = \ddot{Q}\ell = \dot{I}\ell = C_s \ddot{V}_c \ell. \tag{10}$$

Since

$$V_X = \frac{P_{C,\mathrm{rad}}}{I} = \frac{P_{C,\mathrm{rad}}}{\dot{Q}} = \frac{P_{C,\mathrm{rad}}}{\dot{V}_c C_s}, \tag{11}$$

the following nonlinear differential equation replaces Eq. (6):

$$\ddot{V}_c^2 + \frac{1}{K_C C_s}\dot{V}_c V_c = 0. \tag{12}$$

Equation (12) differs from Eq. (6) by having a second-order instead of a third-order derivative and also by the new constant K_C:

$$K_C = \frac{\ell^2}{6\pi\epsilon_0 c^3}. \tag{13}$$

The rest of the proof for the radiation energy follows steps similar to Ref. 1, but in view of the differences we reproduce them here for completeness. Equation (12) has particular solutions (apart from the trivial case $s=0$) given by

$$V_c = -V_{1,0}\exp(st), \tag{14}$$

where

$$s = \left(\frac{1}{K_C C_s}\right)^{1/3}\exp\left(i\frac{(2n+1)\pi}{3}\right) \quad (n=0,1,2). \tag{15}$$

Only the $n=1$ solution (for real V_c) is of physical significance. Here $V_{1,0}$ is the initial voltage on the capacitor C_1 prior to closing the switch.[1]

The calculation of the total radiation energy requires the evaluation of the integral:

$$\begin{aligned} W_{\mathrm{rad}}^C &= \int_{r/c}^{\infty} P_{C,\mathrm{rad}}\,dt \\ &= K_C \int_{r/c}^{\infty} \frac{1}{K_C C_s} C_s^2 V_{1,0}^2 \left(\frac{1}{K_C C_s}\right)^{1/3} \\ &\quad \times \exp\left[-2\left(\frac{1}{K_C C_s}\right)^{1/3}\left(t-\frac{r}{c}\right)\right]dt \\ &= \frac{1}{2}C_s V_{1,0}^2 = \frac{1}{2}\frac{C_1 C_2}{C_1+C_2}V_{1,0}^2. \end{aligned} \tag{16}$$

As we can see, the final result is identical to Ref. 1 and fully accounts for the missing energy in the textbook Kirchoff circuit. For the point dipole we now write down in a way similar to Ref. 1 the equivalent radiation resistance from the Ohm's law relation $V_X = R_{C,\mathrm{rad}} I$, so that

$$R_{C,\mathrm{rad}} = K_C s_2^2 = \frac{K_C^{1/3}}{C_s^{2/3}}, \tag{17}$$

$$s_2 = -\left(\frac{1}{K_C C_s}\right)^{1/3}. \tag{18}$$

We are now in a position to return to the model of Ref. 1 and now include the wire loop (unshielded) so that both the wire loop and capacitors are radiation sources. The questions now become will electromagnetic radiation account for all the missing energy as before and if so, how is this energy partitioned between the capacitors and the wire loop as antennas? Before we answer these questions, it is worth noting that the point dipole model is an extreme limit of the capacitor. Like the corresponding short current-carrying wire dipole[6–11] (as in the case of a long wire[11] of order λ), the contributions should be added vectorially for each point dipole element and then integrated over the capacitance area for a λ size capacitor. The resultant radiation pattern and impedance properties will differ from that of a point dipole. These considerations are given in Appendix A for comparison and reference.

IV. WIRE LOOP WITH CAPACITOR RADIATORS

We now consider the more complicated gedanken experiment in which both capacitors and wire loops are allowed to radiate. There is one further approximation we need to make progress. We assume that both elements do not interact as radiators.[22] In this case, the power radiated by the capacitor point dipole remains as $P_{C,\mathrm{rad}}$ as given by Eq. (9) and the wire magnetic dipole $P_{I,\mathrm{rad}}$ by Eq. (1) as before. The circuit element X now consists of a sum $X=X_I+X_C$ and the voltage drop $V_X=V_{X_I}+V_{X_C}$, where

$$V_{X_I}=K_I\frac{\ddot{I}^2}{I}, \tag{19a}$$

$$V_{X_C}=K_C\frac{\dot{I}^2}{I} \tag{19b}$$

as before. The assumption of the constancy of the current through the system with $I=C_s\dot{V}_c$ yields the third nonlinear differential equation in our study:

$$\dddot{V}_c^2+\frac{K_C}{K_I}\ddot{V}_c^2+\frac{1}{K_IC_s}\dot{V}_cV_c=0. \tag{20}$$

Once again we can employ the ansatz, Eq. (14), which now yields (after eliminating the irrelevant $s=0$ solution as before) a quintic equation of the form:

$$f(s)=s^5+\frac{K_C}{K_I}s^3+\frac{1}{K_IC_s}=0. \tag{21}$$

Equation (21) has only one real root (which must be negative) plus two complex conjugate pairs. We can see this from the positivity of all the coefficients [and hence $f(0)>0$ while $f(-\infty)<0$] as well as the fact that the derivative $f'(s)$ is always positive on the real axis $(s<0)$. None of the complex roots are physically admissible. We do not need to obtain the explicit real solution in order to answer some of the questions posed at the end of Sec. III. We only need to know that there exists only one real root $s=-s_0$ and that it is negative. This fortunate result is due to the following energy partition theorem which we derive.

Energy partition theorem. We first remember that with the ansatz Eq. (14), the power factors are given by

$$P_{I,\mathrm{rad}}=K_IC_s^2\dddot{V}_C^2=K_IC_s^2s^6V_{1,0}^2e^{2st}, \tag{22a}$$

$$P_{C,\mathrm{rad}}=K_CC_s^2\ddot{V}_C^2=K_CC_s^2s^4V_{1,0}^2e^{2st}. \tag{22b}$$

We next multiply Eq. (21) by $K_IC_s^2sV_{1,0}^2e^{2st}$, integrate over time, and substitute the real root $s=-s_0$ to find

$$K_IC_s^2V_{1,0}^2\int_0^\infty s_0^6e^{-2s_0t}dt+K_cC_s^2V_{1,0}^2\int_0^\infty s_0^4e^{-2s_0t}dt$$
$$-C_sV_{1,0}^2\int_0^\infty s_0e^{-2s_0t}dt=0. \tag{23}$$

The last term on the left-hand side of Eq. (23) is the missing energy term and the sum of the first two terms is the total radiated energy. We have thus shown that in this more realistic circuit with capacitor radiation included in the model of Ref. 1, electromagnetic radiation can account for all the missing energy as before. Moreover, after evaluating the integrals in Eq. (23), we obtain the energy partition theorem:

$$\tfrac{1}{2}K_IC_s^2V_{1,0}^2s_0^5+\tfrac{1}{2}K_CC_s^2V_{1,0}^2s_0^3=\tfrac{1}{2}C_sV_{1,0}^2, \tag{24}$$

which shows that the radiation energy is partitioned in the ratio of the radiation resistances $R_{I,\mathrm{rad}}=K_Is_0^4$ and $R_{C,\mathrm{rad}}=K_Cs_0^2$ given by

$$\kappa(-s_0)=\frac{R_{I,\mathrm{rad}}(-s_0)}{R_{C,\mathrm{rad}}(-s_0)}. \tag{25}$$

Equation (25) constitutes an important result of the energy partition theorem. We note that the radiation resistances $R_{I,\mathrm{rad}}(-s_0)$ and $R_{C,\mathrm{rad}}(-s_0)$ are explicitly dependent on the particular solution $s=-s_0$ of Eq. (21). This solution generally differs from s_1 and s_2 given by Eqs. (8) and (18), respectively, because the latter are solutions of different differential equations. Nevertheless, we see that we can obtain some reasonably good bounds without an explicit solution of Eq. (21). First let us define the generally positive definite energy partition function,

$$\kappa(s)=\frac{R_{I,\mathrm{rad}}(s)}{R_{C,\mathrm{rad}}(s)}=\frac{K_I}{K_C}s^2, \tag{26}$$

for arbitrary s. If we substitute Eq. (26) into Eq. (21), we can recast the latter in the following form:

$$\kappa(s)=\left(-\frac{1}{K_CC_ss^3}-1\right). \tag{27}$$

Equation (27) can only be satisfied for a finite number of s values which are the roots of Eq. (21). Nevertheless we can exploit the function defined by the right-hand side (RHS) of Eq. (27). We shall refer to this function as the constrained $\kappa(s)$ while the function defined by the RHS of Eq. (26) shall be referred to as the unconstrained $\kappa(s)$, to avoid having to introduce extra notations. Both functions generally contain different information, but they must coincide at the roots of Eq. (21). Unless otherwise stated, $\kappa(s)$ [and $\chi(s)$ to be introduced later] will be understood to be the constrained function by default.

A plot of the constrained function $\kappa(s)$ is a powerful way to visualize the significance of the radiation and also to look for where the root, $-s_0$, must lie, because the physical regions require $\kappa(s)\geqslant 0$. For instance, we note that the derivative of Eq. (27),

$$\kappa'(s)>0, \tag{28}$$

for all s including the solution $s=s_3=-s_0$. This result is important and is opposite to the unconstrained function Eq. (26). We note that the minimum value of κ is given by $\kappa(s_2)=0$ and the maximum value is $\kappa(0)=\infty$. Hence the

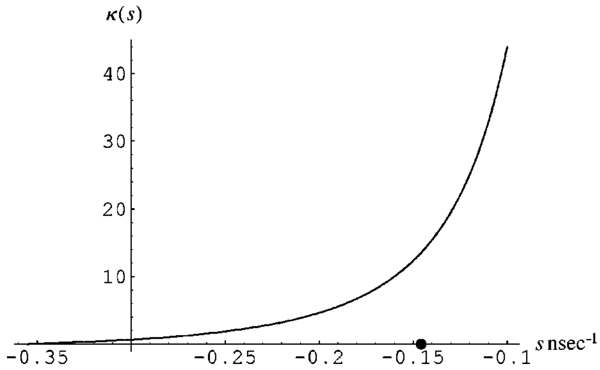

Fig. 2. Plot of $\kappa(s)$ vs s for the usual wire loop. The dark spot on the s axis denotes the value of $s_1 = -(K_1 C_s)^{-1/5}$, which is very close to the exact root (see the text).

root s_3 must be between $[s_2, 0]$ (see Fig. 2). We will be tempted to estimate $\kappa(s_3)$ from $\kappa(s_1)$, because it is an increasing function of s from s_2 to the origin [see Eq. (28) and Fig. 2]. Note also that $s_2 < s_1$ for the cases we consider, due to the point dipole limit $\ell \ll b$. We see that $\kappa(s_1)$ is in fact quite an accurate guess for $\kappa(s_3)$ as follows.

If we now look again at the function $f(s)$ in Eq. (21) whose derivative is positive definite, we have $f(s_2) = -(K_I C_s)^{-5/3} < 0$ and $f(0) = (K_I C_s)^{-1} > 0$. If we note that $f(s_1) = -(K_C/K_I)(K_I C_s)^{-3/5} < 0$ and $f'(s) > 0$, we see that the exact root must be in the interval $[s_1, 0]$. We can do better by successively halving the interval to $[s_1, \frac{1}{2}s_1]$ because $f(\frac{1}{2}s_1) > 0$ and then to $[s_1, \frac{3}{4}s_1]$ and so on until the function $f(s)$ is negative again. Thus the estimate $\kappa(s_1)$ is a lower bound while $\kappa(0.95 s_1)$ is an upper bound in terms of the significance of the capacitor radiation.

In Table I we tabulate these estimated values of $\kappa(s_3)$ for the wire loop, in which we vary the value of the capacitance C_s. For completeness, we have provided the exact value of κ derived from a numerical solution of Eq. (21), showing that the lower bound is quite accurate.

We note that capacitor radiation constitutes about 6.8% of the wire loop for $C_s = 100\,\mu$F to about 17.1% for $C_s = 1000\,\mu$F. Clearly the significance of capacitor radiation increases with C_s, even in the point dipole limit. Thus the neglect of capacitor radiation in a realistic model will require some special arrangements, such as the Faraday cages introduced in Fig. 1.

Table I. Bounds for the ratio of radiation resistances $\kappa(s_3)$ vs the capacitance C_s for the usual wire loop, compared to the exact value obtained from numerical solutions of Eq. (21).

C_s (μF)	κ_{lower}	κ_{exact}	κ_{upper}
100	13.558	13.618	15.980
200	10.033	10.093	11.868
300	8.381	8.441	9.941
400	7.361	7.421	8.752
500	6.647	6.707	7.920
600	6.109	6.170	7.292
700	5.684	5.744	6.796
800	5.337	5.397	6.391
900	5.045	5.105	6.051
1000	4.795	4.856	5.760

V. SELF-INDUCTANCE

We now improve our gedanken experiment further by considering the self-inductance of the loop L.[19] This consideration leads to

$$\dddot{V}_c^2 + \frac{K_C}{K_I}\ddot{V}_c^2 + \frac{L}{K_I}\ddot{V}_c\dot{V}_c + \frac{1}{K_I C_s}\dot{V}_c V_c = 0, \tag{29}$$

which can be treated as before. The modified quintic equation becomes

$$f(s) = s^5 + \alpha s^3 + \beta s^2 + \gamma = 0, \tag{30}$$

where for convenience we define $\alpha = K_C/K_I$, $\beta = L/K_I$, and $\gamma = 1/(K_I C_s)$. Once again the physically admissible real solutions must be negative. Although the derivative $f'(s)$ is no longer positive definite, it is nevertheless monotonic [because $f''(s) = 0$ does not contain any real solutions] and it is a principal cubic (i.e., one that does not contain a quadratic term).[23] The turning points of $f(s)$ is thus determined by

$$f'(s) = s^3 + \tfrac{3}{5}\alpha s + \tfrac{2}{5}\beta = 0, \tag{31}$$

which admits only one real negative solution at $s = x_0$, where

$$x_0 = \frac{-5^{1/3}\alpha + (-5\beta + \sqrt{5}\sqrt{\alpha^3 + 5\beta^2})^{2/3}}{5^{2/3}(-5\beta + \sqrt{5}\sqrt{\alpha^3 + 5\beta^2})^{1/3}}. \tag{32}$$

Since $f'(s) = 0$ has only one root, and therefore $f(s)$ has only one turning point, then $f(s) = 0$ can have at most two negative real roots. Two real roots are impossible for a quintic because the remaining complex roots must come in conjugate pairs. Hence we are left with only one real negative root $(s = s_4 = -s_0)$ as before, which must satisfy the energy partition theorem:

$$\tfrac{1}{2}K_I C_s^2 V_{1,0}^2 s_0^5 + \tfrac{1}{2}K_C C_s^2 V_{1,0}^2 s_0^3 - \tfrac{1}{2}L C_s^2 V_{1,0}^2 s_0^2 = \tfrac{1}{2}C_s V_{1,0}^2. \tag{33}$$

The negative sign on the inductor energy requires an interpretation. For now we assume that the negative sign implies that some of the missing energy is being stored in the inductor. We can define the unconstrained energy partition function $\chi(s)$ as:

$$\chi(s) = \frac{K_I s^4 + K_C s^2}{-Ls}, \tag{34}$$

which compares the ratio of the total radiated energy to the energy transferred to the inductor.[24] If we use Eq. (34), we can do some simple manipulations to show that Eq. (30) now implies that the constrained function $\chi(s)$ is given by

$$\chi(s) = 1 + \frac{1}{LC_s s^2}, \tag{35}$$

which shows that $\chi(s)$ is a positive definite function, whose derivative also is positive for $s < 0$.

We can see the power of plotting the constrained function in Eq. (35) instead of the unconstrained function in Eq. (34). We have already derived an important result for electromagnetic compatibility (EMC),[16] because the minimum value of χ in this case is $\chi(-\infty)$, which is unity. Thus at least half the missing energy must be radiated.[25] Although the lower bound for $\chi(-s_0)=1$, we can easily improve on this bound by noting that $f''(0)>0$ at the trivial turning point $s=0$, that is, $f(0)$ is a local minimum. Also at the turning point $s=x_0$, we have $f''(x_0)=15x_0^3+3\alpha x_0<0$, that is, $f(x_0)$ is a maximum. The effect of the quadratic term in Eq. (30) is to move the root $-s_0$ away from the origin toward s_1 and beyond depending on the magnitude of L. For sufficiently large L the root can even move beyond s_2, which is in fact our case. We can estimate $\chi(-s_0)$ using the turning point $s=x_0$ as an upper bound which improves on the lower bound of unity. However, this bound is extremely close to unity because $\chi(x_0)=1+3.704\times10^{-10}$ for the usual wire loop. These considerations are supported by numerical examples as in Ref. 1, where it was found that for $L=1\ \mu$H, $C_s=0.5\ \mu$F, $R=0$, $K_C=0$, and $b=5$ cm, the root $s_0=18.712\ (\text{ns})^{-1}$, and hence $\chi(-s_0)=1+(5.712\times10^{-9})$. For completeness we give the values for the usual wire loop parameters with $C_s=100\ \mu$F, and we find numerically that $s_0=12.665\ (\text{ns})^{-1}$ and hence $\chi(-s_0)=1+(2.011\times10^{-10})$. Although inconsequential for the usual wire loop, the bound $\chi(x_0)$ will be useful for smaller values of L.

The alert reader might have spotted some difficulties for the case $L\neq0$. These difficulties arise because our ansatz Eq. (14) fails to satisfy the initial boundary condition $I(0)=0$, since with an inductor present, the current cannot change abruptly. Given that there is only one solution for s, we are further unable to construct a linear combination of solutions that will give the correct boundary conditions, quite unlike the standard *LCR* circuit.[2,3,6] Our solution therefore entails a further assumption of overdamping by radiation in which the initial current will appear to be discontinuous over time scales τ that are much greater than a few L/R_{rad}. This time τ is the order of a few microseconds in our case for the parameters we have used.

Another difficulty is that our inductor ends up with a net energy which we earlier interpreted as stored energy. This behavior is unlike the simple *LCR* circuit[2,3,6] (see also Appendix B), and it is impossible in a transient situation. Thus Eq. (14) is not a solution for the transient switching problem. If there is more than one real root present (see Sec. VI), we may hope that our difficulties can be resolved by an appropriate linear combination of these solutions as in the *LCR* circuit.[2,3,6] Unfortunately, we will see that such a linear combination does not provide a solution either. The unrealistic behavior appears to be yet another idealization associated with the point dipole approximation (see Appendix B).

VI. SELF-INDUCTANCE PLUS RESISTANCE

For the bare 5-cm wire loop, the wire resistance is in general quite negligible, $R_w=1.4\ \text{m}\Omega$, so that the analysis of Sec. V suffices. For the case $K_C=0$ some numerical results have been presented in Ref. 1 for various values of additional resistance R corresponding to the underdamped $R\ll R_{\text{cr}}=\sqrt{4L/C_s}$ and overdamped $R\gg R_{\text{cr}}$ cases. The latter reduces to a conventional *LCR* circuit for which radiation is suppressed. However, exactly how this suppression takes place is somewhat obscure from a numerical solution. Hence the calculations in this section will help supplement earlier work[1,3,4] through the use of the constrained energy partition function.

We are especially interested in the minimum value of R such that radiation is suppressed.[26] An analysis of the function

$$f(s)=s^5+\alpha s^3+\beta s^2+\rho s+\gamma=0, \tag{36}$$

with $\rho=R/K_I$ is quite complicated. However, we shall see that it can have at most three negative real roots, and thus is essentially a cubic. We observe that $f'(s)$ is a fourth-order polynomial with $f'(0)>0$ and $f'(-\infty)>0$, but $f''(s)$ is a cubic that can have only one negative real root λ_0, for which $f'(\lambda_0)$ is a minimum. Thus if $f'(\lambda_0)>0$, then $f(s)$ has no turning points, that is, $f(s)$ has at most one root. If $f'(\lambda_0)<0$, then $f(s)$ has two turning points and hence it can have at most three roots. However, not all real negative roots are of equal importance to radiation.

The energy partition function is an extremely powerful tool, even an analysis of the quartic function $f'(s)$ is nontrivial. To proceed further, we will need some nontrivial generalizations of the energy partition theorem for the multi-root case, which unfortunately we have not been able to solve (see Appendix B). Nevertheless, we continue as before, bearing in mind that our solution will not satisfy the transient boundary conditions at $t=0$.

We can by analogy with the previous cases define the unconstrained $\chi(s)$ as:

$$\chi(s)=\frac{s^4+\alpha s^2}{-\beta s+\rho}=\frac{K_I s^4+K_C s^2}{-Ls+R}, \tag{37}$$

and again from the associated fifth-order equation we derive the constrained function as:

$$\chi(s)=-\left(\frac{\beta(s-w_1)(s-w_2)}{-\beta s^2+\rho s}\right)=-\left(\frac{Ls+R+\dfrac{1}{C_s s}}{-Ls+R}\right), \tag{38}$$

where w_1 and w_2 are the roots of the *LCR* circuit:

$$w_{1,2}=-\frac{\rho}{2\beta}\left(1\pm\sqrt{1-\frac{4\beta\gamma}{\rho^2}}\right)$$
$$=-\frac{R}{2L}\left(1\pm\sqrt{1-\frac{4L}{C_sR^2}}\right). \tag{39}$$

The negative sign in Eq. (38) is very important because it shows that there are regions of s that are physically unimportant to radiation. We have for $s\to0$, $\chi(s)\to\infty$; also $\chi(s)\to1$ for $s\to-\infty$ just as in Eq. (35). However, there is now a turning point for $\chi(s)$ given by

$$\zeta=\frac{-\sqrt{L}-\sqrt{L+2C_sR^2}}{2C_s\sqrt{L}R}, \tag{40}$$

which is a minimum (see Fig. 3). Thus in the overdamped case $R>R_{\text{cr}}$ we have a range $w_1<s<w_2$ where $\chi(s)<0$ lies in an unphysical region in which radiation may be eliminated if the root moves into this range. In the underdamped case this possibility does not exist. However, radiation cannot be completely suppressed even for $R_{\text{cr}}<R<R_m$. Our problem is to determine R_m and to see how $\chi(s)$ behaves as $R\to R_m$. To do this we exploit the further use of $\chi(s)$ as follows.

We first write Eq. (36) in the form:

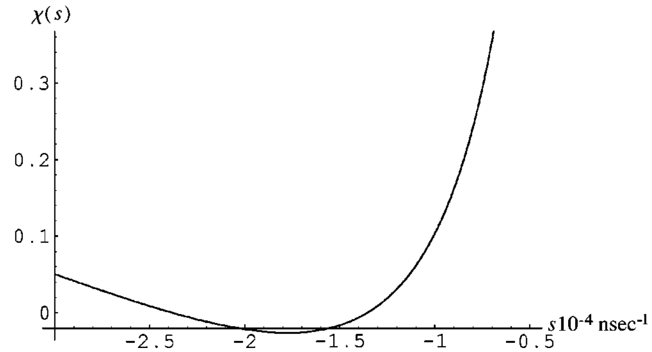

Fig. 3. Plot of $\chi(s)$ vs s for the usual wire loop when it is just overdamped with $R=1.04R_{\mathrm{cr}}$. We show that in this case, $\chi(s)<0$ near its turning point.

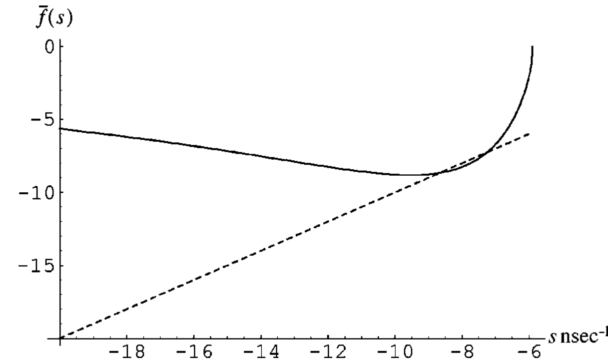

Fig. 4. Plot of $\overline{f}(s)$ vs s for the usual wire loop when it is heavily overdamped with $R=1.64\times 10^4 R_{\mathrm{cr}}$. We see that there are two roots when $\overline{f}(s)$ intersects the line of unit slope (dotted) [see Eq. (43)].

$$f(s)=s^3 g(s)+\beta(s-w_1)(s-w_2)=0, \tag{41}$$

where $g(s)=s^2+\alpha$ is a positive definite monotonic function of s. Hence for our purposes it serves merely as a variable positive cubic coefficient, which implies that $f(s)$ is essentially a cubic as noted. From the definition of the constrained function in Eq. (38) we can further rewrite $f(s)$ as:

$$f(s)=s^3 g(s)-s\chi(s)(-\beta s+\rho) \tag{42a}$$

$$=s^2 g(s)-\chi(s)(-\beta s+\rho)=0. \tag{42b}$$

The last step follows because $s=0$ cannot be a solution. Equation (42b) now has the form of a quadratic whose roots are given by

$$s=\overline{f}(s)=\frac{-\beta\chi(s)-\sqrt{\beta^2\chi(s)^2+4\rho g(s)\chi(s)}}{2g(s)}, \tag{43}$$

where the positive square root is inadmissible. Equation (43) is extremely useful for finding the roots, because by plotting $y=s$ and $y=\overline{f}(s)$, the root is easily found from the intersection. This procedure is also numerically very accurate, because the order of magnitude of $\overline{f}(s)$ and s is almost identical in the region of the roots. The same cannot be said of a plot of $f(s)$ vs s. As $\chi(s)$ varies, the roots of Eq. (43) will in general develop from our earlier roots s_4 and w_1. However there is also a third root close to w_2, which for large R moves toward the region where $\chi(s)$ is unphysical, see Eq. (38). These results can be shown by studying the various limits $\rho\to 0$, $g(s)\to 0$, and in particular, the limit $\rho\to\infty$ for which analytical results are obtainable (see the following).

In Fig. 4 we plot $\overline{f}(s)$ for the usual wire loop parameters. The value $R=1.64\times 10^4 R_{\mathrm{cr}}\approx 1826\ \Omega$, where $R_{\mathrm{cr}}\approx 0.111\ \Omega$ is now very close to the minimum value R_m above which radiation becomes insignificant. The roots are found to be $s=-8.663\,(\mathrm{ns})^{-1}$, $-7.253\ (\mathrm{ns})^{-1}$, and $-0.548\times 10^{-8}\ (\mathrm{ns})^{-1}$ for which the values of χ are $\chi=0.190$, 0.104, and 6.348×10^{-10}, respectively. However even for this large value of R, radiation is suppressed but is not negligible.[27] To reduce radiation to a negligible level, the value of R should be closer to $R_m\approx 1.68\times 10^4 R_{\mathrm{cr}}\approx 1871\ \Omega$ so that the only remaining root for s is close to $s=w_2$ for which radiation is negligible. Hence the change occurs over a rather small range of R within about 2.5% of R_m with the unfortunate fact that some 10^4 times the resistance of R_{cr} or 10^6 times the resistance of R_w is required to suppress radiation to a negligible level. This sharp transition behavior differs from the case $L=0$, $K_C=0$ (and hence $R_{\mathrm{cr}}=0$) for which numerical results have already been presented in Ref. 1 (see their Fig. 5). By using the constrained $\chi(s)$ method presented here, we can show analytically from Eq. (43) that $\chi(-s_0)$ (which in that case is equivalent to the ratio of R_{rad}/R) vanishes as R increases near the transition region:

$$\chi(-s_0)=\frac{K_I}{5K_I+R^5 C_s^4}. \tag{44}$$

Thus our results show that the suppression of radiation using a damping resistor R in the more realistic circuit may not be a practical solution for power supply applications.

VII. CAPACITOR ANTENNAS

There have been recent controversies in the engineering community about certain capacitor antennas patented in the U.S. and Great Britain, that purport to use Poynting vector synthesis for their operational principles. The owners of these patents claim that their invention produces the Poynting vector from crossed **E** and **B** fields directly at the source and cancels all near fields.[17,18] These antennas are now commercial products that have produced contradictory results for medium frequency broadcast applications. Although we do not subscribe to the theory of Poynting vector synthesis, our analysis in Sec. VI shows that capacitor radiation is a reality, and therefore a capacitor antenna may be possible. A finite size capacitor antenna constructed from two circular metal plates will (depending on frequencies) have impedance characteristics that differ from a point electric dipole. Such an antenna may be modeled as an appropriate (perhaps quite complicated) magnetic current loop (see Appendix A).[9–11,28] We are now not interested in suppressing the radiation, but in minimizing nonradiative losses. In addition, an antenna needs to focus all of its radiation in the desired operating frequency range, because stray radiation will be a source of EMC problems.

The detailed study of such an antenna is complicated, and we will not pursue it here. A preliminary comparison of radiation efficiency, though, can be obtained by noting that for a sinusoidally driven current, the imaginary switching frequency parameter s can be replaced by the real angular frequency $s\to i\omega$ $(\omega=2\pi f)$.[9–11] In this case the radiation efficiency can be determined by comparing the radiation resistances of the capacitor and the wire loop, if both systems are driven by identical signal sources that are optimally matched for power transfer.[9–11] For a small size capacitor antenna (typically of radius $a \ll \lambda/10$), the ratio of its radiation resistance compared to the small wire loop is now given by [see Eqs. (8) and (18)]

$$\kappa_{C,I} = \frac{K_C \omega^2}{K_I \omega^4} = \frac{\ell^2 \lambda^2}{4\pi^4 b^4}. \tag{45}$$

For the point dipole approximation we require $\ell \ll \lambda$ and $b \ll \lambda$. A practical system would also have $\ell \ll b$ such as $\ell = b/10$ and $b = \lambda/10$. With these parameters, $\kappa_{C,I} = 1/(4\pi^4) \approx 2.57 \times 10^{-3}$, which makes the capacitor a rather poor antenna in comparison to the wire loop.

For larger size systems other considerations become important, which we will not discuss here (see Appendix A). Wire antennas, for example, resonate at the operating frequency by making use of the free space capacitance leading to the approximate formula:[29]

$$L_{\text{dipole}} \approx \frac{143}{f}, \tag{46}$$

for the length of a thin wire half-wave dipole antenna in meters, with the frequency f in MHz. Equation (46) is accurate up to about 100 MHz. In the same way, capacitor antennas can resonate using the free space inductance. The formula corresponding to Eq. (46) for the capacitor antenna, including its relative performance, would be an interesting research project for a graduate student. For practical systems a lumped-parameter circuit analysis including the proper self-inductance L as well as other stray inductances would be needed.[19] However modern PSpice software[30] and other antenna modeling software[31] do not include the radiation resistance from the capacitors discussed here, so some care needs to be exercised in their use.

VIII. CONCLUSION

We have extended the discussion of the radiation from the transient switching of charges between two capacitors. We have shown that the capacitors themselves can radiate, using a point electric dipole model. We found this radiation to be small but not insignificant, and hence an extension of the lumped-parameter circuit model of Ref. 1 was also presented. We then included the self-inductance and an external resistance R, showing that a minimum value R_m, which must be approximately 10^6 times the wire resistance R_w, is needed to suppress the radiation. The exact value is critical, and we developed an accurate numerical procedure to extract this parameter using the constrained energy partition function $\chi(s)$.

Exactly how much radiation a commercial capacitor radiates will depend on its effective inductance, resistance, and dielectric properties.[32,33] The calculation of the radiation would be a good exercise for an undergraduate student using the methods developed here. Our results show that although the details of the capacitor radiators are unimportant for the recovery of the missing energy, they are important for the study of the transient response and electromagnetic compatibility. Unfortunately, further difficulties remain for the case $L \neq 0$, due to the failure of our solutions to satisfy the boundary conditions required for transient behavior at the initial time (see Appendix B).

Antenna theory is an interesting topic with instructive articles published previously.[34,35] Some of the results obtained in this paper may be useful additions to modern texts on electromagnetism and antenna theory. The implications of our study for EMC directives and for novel antenna designs are topics for further research. Students should be taught to appreciate that there is much more to meet the eye than two simple capacitors.

ACKNOWLEDGMENTS

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APPENDIX A: RADIATION PROPERTIES OF A FINITE SIZE CAPACITOR ANTENNA

In this appendix we provide some preliminary studies of a finite size air-spaced capacitor antenna. The capacitor is assumed to be made of two circular discs of radius a ($a \sim \lambda$) with separation $\ell \ll \lambda$. As far as the author is aware, the elementary results presented here are not found in standard texts.[6–11] The key assumption is that there is a uniform charge area density σ on the disc, that is, $Q = \pi a^2 \sigma$ (see Fig. 5). Just as for case of a large current loop, the ability to maintain a uniform charge density at finite frequencies requires the introduction of a rather sophisticated type of phase shifters,[11] the details of which we will not be concerned with here. For an elementary discussion of why a capacitor's charge density cannot be uniform at high frequencies, see, for example, Ref. 28. In a more sophisticated model, $\sigma(\rho)$ has to be determined self-consistently with the field E, although some approximate charge distribution may suffice as in the wire antenna.

We assume a sinusoidal charge oscillation of the type $q(t) = q_0 \cos \omega(t - r/c)$ for each area element that forms the infinitesimal dipole $dp_z = \sigma \ell \, d\rho \rho \, d\phi$. Then in a plane cutting through a pair of opposite dipole elements, the radiation E field at a distant point P at (r, θ, ϕ) will be a sum of two components given by[10–12]

$$dE_\theta = dE_\theta^0 e^{i\psi/2} + dE_\theta^0 e^{-i\psi/2} = 2 dE_\theta^0 \cos \psi/2, \tag{A1}$$

where dE_θ^0 is the field due to each dipole element,

$$dE_\theta^0 = \frac{\sigma \ell \, d\rho \rho \, d\phi}{4\pi\epsilon_0} \frac{\omega^2}{rc^2} \sin\theta \cos\omega(t - r/c). \tag{A2}$$

The relative phase shift ψ is due to the path difference (see the insert to Fig. 5) and is given by

$$\psi = 2\beta\rho \sin\theta, \tag{A3}$$

where $\beta = \omega/c$. We also can easily obtain the H field in the radiation zone because it is simply given by

$$dH_\phi = \frac{dE_0}{Z_0}, \tag{A4}$$

where $Z_0 = \sqrt{\mu_0/\epsilon_0} \approx 377\ \Omega$ is the impedance of free space. An integration over ϕ from 0 to π is trivial (note a full 2π integration would double count). The integral over ρ from 0 to a is also trivial with the final result:

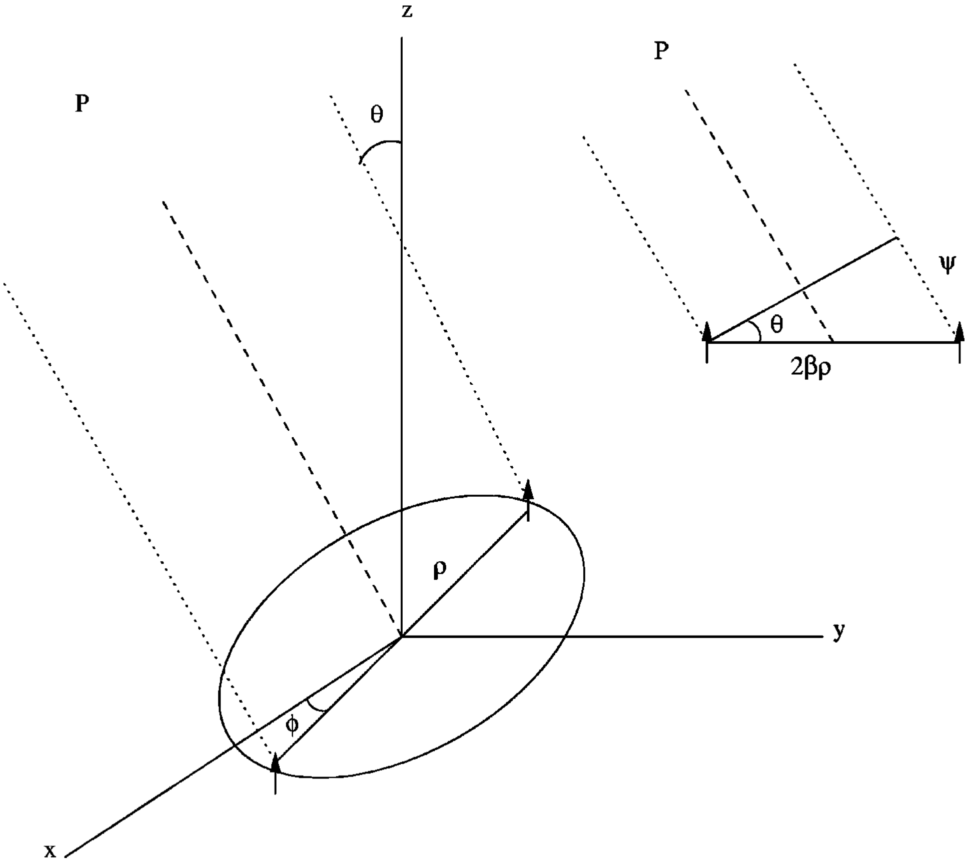

Fig. 5. Radiation fields from a finite size capacitor antenna. The insert shows the path difference between a diametrically opposite pair of dipoles on the disc to a distant point *P*.

$$E_\theta = \frac{\sigma \ell}{2\epsilon_0 r}\left[\beta a \sin(\beta a \sin\theta) + \frac{1}{\sin\theta}\cos(\beta a \sin\theta) - \frac{1}{\sin\theta}\right]\cos\omega(t - r/c). \tag{A5}$$

For $\beta a \to 0$ we easily recover the point dipole limit,

$$E_\theta^0 = \frac{\sigma \ell \omega^2 a^2}{4\epsilon_0 r c^2}\sin\theta \cos\omega(t - r/c). \tag{A6}$$

The radiation pattern of our antenna which can be obtained from Eq. (A5) is $I(\beta a, \theta) = |E_\theta|^2/Z_0$ and can be easily compared with a point dipole, which as can be seen in Eq. (A6) is proportional to $\sin^2\theta$. Unfortunately, calculating the total radiated power involves difficult integrals over θ, and we will not pursue this calculation further. The calculation of the self-impedance would be even more complicated than that for the wire antenna.[9–12,15]

APPENDIX B: THE MULTI-ROOT CASE AND DIFFICULTIES WITH TRANSIENT BOUNDARY CONDITIONS

If we assume that we have three real roots $s = \xi_i$, $i = 1,2,3$ (remember that we can only have either one or three) for Eq. (36), we can construct a linear combination from the solutions as:

$$V_c(t) = Ae^{\xi_1 t} + Be^{\xi_2 t} + Ce^{\xi_3 t}. \tag{B1}$$

The appropriate boundary conditions are $V_c(0) = -V_{1,0}$ and $I(0) = C_s \dot{V}_c(0) = 0$. However Eq. (29) is a third-order differential equation. Hence one further boundary condition is necessary, which we may choose as $\dot{I}(0) = C_s \ddot{V}_c(0) = 0$. For comparison, note that the simple *LCR* circuit has only the first two boundary conditions because it has just two roots. The resulting 3×3 matrix is quite involved so we shall not discuss this further. Moreover, we do not need to pursue this calculation to see that it would not work. This procedure fails because Eq. (29) is a nonlinear differential equation. The substitution of the solution Eq. (B1) will produce cross terms that do not cancel (see also Appendix B in Ref. 1). The reader might think that a numerical integration of Eq. (29) could produce the appropriate transient solution. Once again, he/she will be disappointed because the nature of Eq. (29) with the required boundary conditions: $V_c(0) = -V_{1,0}$, $I(0) = C_s \dot{V}_c(0) = 0$ is incompatible with any real value for the higher derivatives (including zero) at $t = 0$. This incompatibility can be seen by examining the Taylor series of $V_c(t)$ near the origin of time $t = 0$. Hence any numerical integration scheme would fail to generate a real solution. These difficulties appear to be due to the point dipole approximation.

Nevertheless, we can easily see that all the missing energy must be dissipated in the resistances $R_{I,\mathrm{rad}}$, $R_{C,\mathrm{rad}}$, and R, as it should be. This result follows once again from the energy partition theorem, because by multiplying Eq. (29) by C_s^2 and integrating, we have

$$C_s^2 \int_0^\infty dt\left(K_I \dddot{V}_c^2 + K_C \ddot{V}_c^2 + L\ddot{V}_c \dot{V}_c + R\dot{V}_c^2 + \frac{\dot{V}_c V_c}{C_s}\right) = C_s^2 \int_0^\infty dt (K_I \dddot{V}_c^2 + K_C \ddot{V}_c^2 + R\dot{V}_c^2) - \frac{1}{2} C_s V_{1,0}^2 = 0. \tag{B2}$$

The second line in Eq. (B2) follows from

$$
\begin{aligned}
E_L &= \int_0^\infty L I \dot{I} dt = \frac{1}{2} L I^2 \big|_0^\infty = 0, \\
E_C &= \int_0^\infty C_s V_c \dot{V}_c dt = \frac{1}{2} C_s V_s^2 \big|_0^\infty ,
\end{aligned}
\tag{B3}
$$

for a proper transient solution in which $I(0)=I(\infty)=0$. This conclusion means that for the case $R=0$ but $L\neq 0$, all the missing energy must be radiated, and our results are therefore inadequate.

In view of these difficulties, a generalization of the constrained partition function for the transient situation when $L\neq 0$ seems to be nontrivial. The analysis shows again that the two-capacitor problem with radiation still remains elusive.

[a]Electronic mail: tuckchoy@ieee.org

[1]T. Boykin, D. Hite, and N. Singh, ''The two-capacitor problem with radiation,'' Am. J. Phys. **70** (4), 415–420 (2002).

[2]See, for example, D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics* (Wiley, New York, 1993), 4th ed. p. 750. See also the extended 6th edition (2001), Problem 27P, p. 609.

[3]R. A. Powell, ''Two-capacitor problem: A more realistic view,'' Am. J. Phys. **47** (5), 460–462 (1979).

[4]K. Mita and M. Boufaida, ''Ideal capacitor circuits and energy conservation,'' Am. J. Phys. **67** (8), 737–739 (1999); ''Erratum: Ideal capacitor circuits and energy conservation,'' **68** (6), 578 (2000).

[5]A. Gangopadhyaya and J. V. Mallow, ''Comment on 'Ideal capacitor circuits and energy conservation' by K. Mita and M. Boufaida [Am. J. Phys. **67** (8), 737–739 (1999)],'' Am. J. Phys. **68** (7), 670–672 (2000).

[6]J. R. Reitz, F. J. Milford, and R. W. Christy, *Foundations of Electromagnetic Theory* (Addison–Wesley, Reading, MA, 1993), 4th ed.

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[9]R. E. Collin, *Antennas and Radio Wave Propagation* (McGraw–Hill, New York, 1985).

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[11]J. D. Kraus, *Antennas* (McGraw–Hill, New York, 1988).

[12]S. A. Schelkunoff, *Electromagnetic Waves* (Van Nostrand, New York 1948).

[13]R. F. Harrington, *Time-Harmonic Electromagnetic Fields* (Wiley-Interscience, New York, 2001), pp. 7–28.

[14]References 8 and 11 considered the magnetic charge dipole, but their results were for the electric current loop.

[15]There also is a brief mention of capacitor antennas but with no explicit calculations in S. A. Schelkunoff, *Advanced Antenna Theory* (Wiley, New York, 1952), pp. 19–22.

[16]Radio frequency interference is regulated in the US by the FCC, see, for example ''The FCC Interference Handbook,'' which can be downloaded from ⟨http://www.kyes.com/antenna/interference/tvibook.html⟩. Parts 0,1,2,15,18,68,76 and 97 of Title 47 of the *Code of Federal Regulations* address interference issues, in particular Part 15, Sec. 15.15, which governs general technical requirements for intentional or unintentional radiators. In Europe the EU member states have adopted (since 1995) Electromagnetic Compatibility (EMC) directives, which require all electronic and electrical equipment sold in the EU to carry a Conformité Européenne (CE) certificate marking.

[17]U.S. Patent 6486846 B1 filed by T. Hart. See also B. Prudhomme, ''The EH antenna—Exceptional or hype,'' Monitoring Times **22** (4), 22 (2003) or ⟨http://www.eh-antenna.com⟩. For a recent positive review of this antenna, see H. R. Henly, ''The *Arno Elettronica* E-H antennas,'' Radio Communications **79** (9), 21–23 (2003), and references therein.

[18]British Patent 9718311 and U.S. Patent 6025813 filed by M. Hately and F. Kabbary. See also P. Hawker, ''Poynting vector synthesis and the CFL,'' Radio Communications **78** (8), 63 (2002), and references therein.

[19]The circuit impedance element X is in general nonlinear and complex (that is, it contains both amplitude and phase) due to the radiation properties of the wire. For small loops and thin wires of radius δ ($\delta/b \ll 1$), the self-inductance $L\approx \mu_0 b(\ln(b/\delta)-7/4)$—Refs. 3 and 12. For the usual wire loop, $L\approx 0.311\ \mu$H (Ref. 3). For larger loops, this self-inductance should be corrected for radiation (Refs. 9–12), but we shall not pursue this here.

[20]The assumption that the displacement current through the capacitors can be equated to the circuit current I for the purpose of calculating the radiation loss is only valid for long wavelengths $\lambda \gg b \gg \ell$ and must be kept in mind for large systems.

[21]L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1987), p. 173.

[22]As with all multi-element antenna problems (Refs. 9–11), this assumption is a zeroth-order approximation only, which is valid for point dipoles, small loops, and long wavelengths. For larger systems and smaller wavelengths, the consideration of interaction among the antenna elements will be necessary, but would take us too far afield into advanced antenna theory and is not necessary for our purposes here.

[23]It can be shown that any arbitrary cubic can be transformed, using a linear transformation, into a principal cubic. The latter can then be solved using trigonometric functions or further transformed into a quadratic, see http://mathworld.wolfram.com/CubicEquation.html.

[24]We use a different symbol now, because some of the missing energy is not radiated.

[25]This nontrivial result is an important consequence of the energy partition theorem in Eq. (33). However all the missing energy should be radiated because there is no other circuit resistance here. Hence we have run into some problems.

[26]For a switched-mode power supply, energy transfers between capacitors and inductors are an integral part of its operation. To comply with EMC regulations, it seems that the power supply engineer must compromise between power lost through adding an external resistor R vs radiation. In cases, where this loss is unacceptable, shielding remains the only option.

[27]The exact fraction of the missing energy radiated cannot be calculated at this stage due to problems with the initial boundary conditions.

[28]An excellent discussion of how a real capacitor behaves at finite frequencies can be found in R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison–Wesley, Reading, MA, 1964), Vol. II, Chap. 23.

[29]The difference in this physical length from an exact half-wave length has to do with the self-inductance (Ref. 19). An exact half-wave length dipole in fact turns out to be reactive, that is, has an impedance of $Z\approx 73\Omega + i42.5\Omega$; hence a shortening of the length is required to achieve resonance (Refs. 9–11).

[30]A number of semiconductor manufacturers now provide free Spice simulation software, such as Linear Technology, which can be downloaded from http://www.linear-tech.com/software. More traditional DOS shareware versions such as WINSPICE1.03 can also be obtained from http//www.willingham2.freeserve.co.uk/winspice.html.

[31]Affordable antenna modeling software, with excellent free demo versions such as EZNEC 3.0, a PC version of the earlier FORTRAN based Numerical Electromagnetic Codes (NEC), is available from http://eznec.com.

[32]J. M. Herbert, *Ceramic Dielectrics and Capacitors* (Gordon and Breach, New York, 1985).

[33]C. Bateman, ''Understanding capacitors—Aluminum and tantalum,'' Electronics World **104**, 495–497 (1998).

[34]Glenn S. Smith, ''Teaching antenna reception and scattering from a time-domain perspective,'' Am. J. Phys. **70** (8), 829–844 (2002).

[35]Reuben Benumof, ''The receiving antenna,'' Am. J. Phys. **52** (6), 535–538 (1984).